APPLIED MATH How Wavelets Allow Researchers to Transform, and Understand, Data Built upon the ubiquitous Fourier transform, the mathematical tools **15**

known as wavelets allow unprecedented analysis and understanding of continuous signals.

n an increasingly data-driven world, mathematical tools known as

wavelets have become an indispensable way to analyze and

understand information. Many researchers receive their data in the

information evolving over time, such as a geophysicist listening to

form of continuous signals, meaning an unbroken stream of

sound waves bouncing off of rock layers underground, or a data scientist studying the electrical data streams obtained by scanning images. These data can take on many different shapes and patterns,

Ana Kova/Quanta Magazine

making it hard to analyze them as a whole or to take them apart and study their pieces — but wavelets can help. Wavelets are representations of short wavelike oscillations with different frequency ranges and shapes. Because they can take on many forms — nearly any frequency, wavelength and specific shape is possible — researchers can use them to identify and match specific wave patterns in almost any continuous signal. Because of their wide versatility, wavelets have revolutionized the study of complex wave phenomena in image processing, communication and scientific data streams. "In fact, few mathematical discoveries have influenced our

technological society as much as wavelets have," said Amir-Homayoon Najmi, a theoretical physicist at Johns Hopkins University. "Wavelet theory has opened doors to many applications in a unified framework with an emphasis on speed, sparsity and accuracy that were simply unavailable before." Wavelets came about as a kind of update

allows researchers to split a signal promising ideas in science and stream into its constituent parts, mathematics. Journey with us enabling, for instance, a seismologist to and join the conversation. identify the nature of underground structures based on the intensity of the different frequencies in reflected sound waves. As a result, the Fourier transform has led directly to a number of applications in scientific research and technology. But wavelets allow for much more precision. "Wavelets have opened the door to many improvements in de-noising, image restoration and image analysis," said Véronique Delouille, an applied mathematician and astrophysicist at the Royal Observatory of Belgium who uses wavelets for analyzing images of the sun.

That's because Fourier transforms have a major limitation: They only

Hungarian physicist who in 1946 suggested cutting the signal into

short, time-localized segments before applying Fourier transforms.

However, these were difficult to analyze in more complicated signals

geophysical engineer Jean Morlet to develop the use of time windows

to investigate waves, with the windows' lengths depending on the

frequency: wide windows for low-frequency segments of the signal

"

There are many kinds of wavelets, and you can squish

them, stretch them, you can adapt them to the actual

with strongly changing frequency components. This led the

and narrow windows for high-frequency segments.

to an enormously useful mathematical

discovered that any periodic function —

cyclically — could be expressed as the

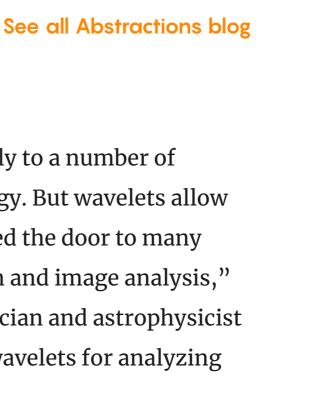
sum of trigonometric functions like sine

and cosine. This proved useful because it

technique known as the Fourier

transform. In 1807, Joseph Fourier

an equation whose values repeat



Abstractions navigates

supply information about the frequencies present in a signal, saying nothing about their timing or quantity. It's as if you had a process for determining what kinds of bills are in a pile of cash, but not how many of each there actually were. "Wavelets definitely solved this problem, and this is why they are so interesting," said Martin Vetterli, the president of the Swiss Federal Institute of Technology Lausanne. The first attempt to fix this problem came from Dennis Gabor, a

image you are looking at. " Daan Huybrechs, Catholic University of Leuven But these windows still contained messy real-life frequencies, which were hard to analyze. So Morlet had the idea of matching each segment with a similar wave that was mathematically well understood. This allowed him to grasp the overall structure and timing of these segments and explore them with much greater accuracy. In the early 1980s Morlet named these idealized wave patterns "ondelettes,"

French for "wavelets" — literally, "little waves" — because of their

centered around a specific wavelength and analyzed by being paired

with the matching wavelet. Now faced with a pile of cash, to return to

appearance. A signal could thus be cut up into smaller areas, each

the earlier example, we'd know how many of each kind of bill it

Roughly, imagine that you slide a particular wavelet, of a specific

particularly good match, a mathematical operation between them

known as the dot product becomes zero, or very close to it. By scanning

Research on wavelets evolved quickly. The French mathematician Yves

waiting for his turn at a photocopier when a colleague showed him a

the entire signal with wavelets of different frequencies you can piece

frequency and shape, over the raw signal. Whenever you have a

together a solid picture of the entire signal train, allowing for a

Meyer, a professor at the École Normale Supérieure in Paris, was

contained.

thorough analysis.

paper on wavelets by Morlet and the theoretical physicist Alex Grossmann. Meyer was immediately fascinated and took the first available train to Marseille, where he began working with Grossman and Morlet, as well as the mathematician and physicist Ingrid Daubechies (now at Duke University). Meyer would go on to win the Abel prize for his work on wavelet theory. A few years later, a graduate student at Pennsylvania State University studying computer vision and image analysis named Stéphane Mallat bumped into an old friend at the beach. The friend, a graduate student with Meyer in Paris, told Mallat about their research in wavelets. Mallat understood the importance of Meyer's work for his own research right away, and quickly teamed up with Meyer. In 1986 they produced a paper on the application of wavelets in image analysis.

Ultimately, this work led to the development of the JPEG2000, a form

of image compression in use around the world. The technique analyzes

the signal of a scanned image with wavelets to produce a collection of

pixels that is overall much smaller than the original image while still

allowing the reconstruction of the image with the original resolution.

This technique proved valuable when technical constraints limited the

transmission of very large data sets.

Wavelet Zoo

Symlet

By drawing from different "families" of wavelets, with different individual shapes and frequency structures, researchers can analyze virtually any kind of stream of continuous data. **Daubechies Mexican Hat** Coiflet Morlet **Beylkin** Haar

Meyer

Battle-Lemarie

Samuel Velasco/Quanta Magazine Part of what makes wavelets so useful is their versatility, which allow them to decode almost any kind of data. "There are many kinds of wavelets, and you can squish them, stretch them, you can adapt them to the actual image you are looking at," said Daan Huybrechs, a mathematical engineer at the Catholic University of Leuven in Belgium. The wave patterns in digitized images can differ in many aspects, but wavelets can always be stretched or compressed to match sections of the signal with lower or higher frequencies. The shapes of wave patterns can also change drastically, but mathematicians have developed different types, or "families," of wavelets with different wavelength scales and shapes to match this variability. **RELATED:** Yves Meyer, Wavelet Expert, Wins Abel Prize In Noisy Equations, One Who Heard Music 3. Latest Neural Nets Solve World's Hardest Equations Faster Than Ever Before One of the best-known wavelet families is the Daubechies mother wavelet, whose members have a self-similar fractal structure, with large asymmetrical peaks mimicking smaller replications of the peaks. These wavelets have proved so sensitive to image analysis that experts have used them to distinguish original Vincent van Gogh paintings from forgeries. Other wavelet families known for their shapes include the Mexican hat, with one central maximum and two adjacent minima,

and the Coiflet wavelet (named after the mathematician Ronald

peaks instead of flat zones. These are useful for capturing and

streams generated by scientific instruments.

kind of data it is.

Correction: October 14, 2021

Coifman at Yale University), similar to the Mexican hat but with sharp

eliminating unwanted noise spikes in images, sound signals and data

Besides their use in analyzing sound signals and in image processing,

discover patterns in scientific data by allowing them to analyze entire

are," said Huybrechs. "There is something about wavelets that makes

them the 'right' way to look at data," and that's true no matter what

An earlier version of the infographic in this article misspelled the Battle-

Lemarie wavelet family, and used a less common spelling of Beylkin.

data sets at once. "It always strikes me how diverse the applications

wavelets are also a tool in basic research. They can help researchers

Alexander Hellemans Contributing Writer October 13, 2021 Abstractions blog applied math explainers data mathematical physics mathematics All topics →

Share this article

Subscribe now

Recent newsletters →

Subscribe

A Hint of Dark Matter Sends Physicists

By JONATHAN O'CALLAGHAN | OCTOBER 19, 2021 | 2 |

Neuron Bursts Can Mimic Famous AI

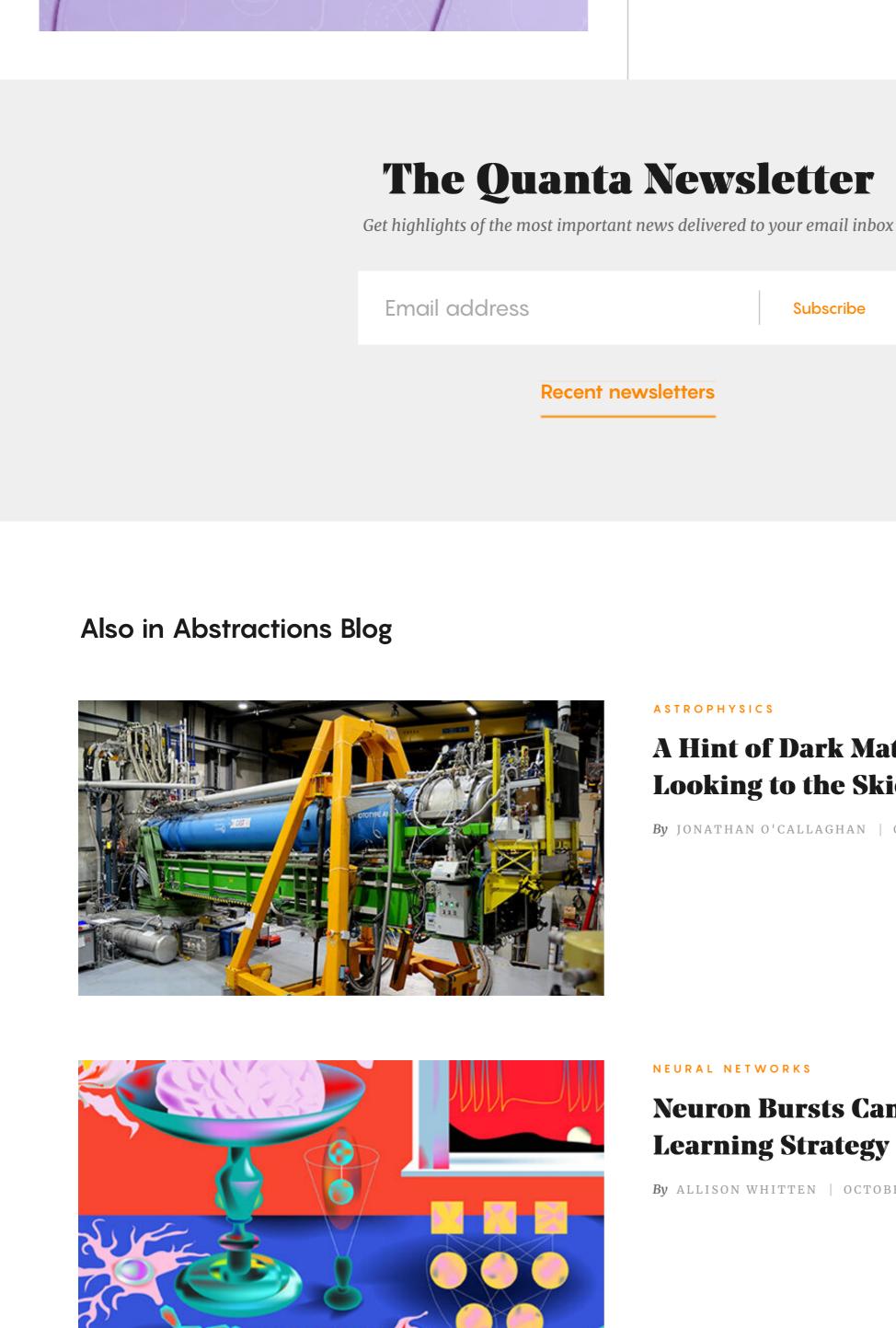
By ALLISON WHITTEN | OCTOBER 18, 2021 | 1 |

ASTROPHYSICS

NEURAL NETWORKS

Learning Strategy

Looking to the Skies



Entangled

with Quanta's

Science Podcasts

NOBEL PRIZE **Chemistry Nobel Prize Honors**

Technique for Building Molecules

By JORDANA CEPELEWICZ | OCTOBER 6, 2021 | 3 |



NEXT ARTICLE

Quantamagazine About Quanta | Archive | Contact Us | Terms & Conditions | Privacy Policy | Simons Foundation All Rights Reserved © 2021

The Astronomer Who's About to See the Skies of Other Earths

Show comments

(y) (a) (b)